1/f Fluctuations in a Non-Hyperbolic System(6) Approaches from mathematical science and quantum information, Chaos and Nonlinear Dynamics in Quantum-Mechanical and Macroscopic Systems)

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1/f Fluctuations in a Non-Hyperbolic System

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A new dynamical system which is non-hyperbolic and area-preserving is introduced. Through numerical simulation, it is found that this system exhibits $1/f^{\gamma}$ fluctuations and the value of the exponent γ which is obtained numerically is in a good agreement with the theoretical prediction.

区分線型写像をうまく構成することで、非双曲的な面積保存写像を構成できることを指摘し、その写像の拡大方向のダイナミクスについて、$1/f^{\gamma}$ ゆらぎが表れることが数値的に示す。また、この指数 γ は時間発展演算子の固有値解析によって得られた結果に良く一致することが分かった。

1 Model

The dynamical system we consider here is a piecewise linear map shown in Fig. 1. This map $\phi(x) : [0,1] \to [0,1]$ consists of the infinite number of the linear segments; the circles plotted in Fig. 1 indicate endpoints of the linear segments. This 1-dimensional map $\phi(x)$ can be approximated as follows: $\phi(x) \simeq x + C_- x^{\frac{\beta}{\beta-1}}$ when $x \searrow 0$, while $\phi(x) \simeq C_+(x-1/2)^{\frac{\beta-1}{\beta}}$ when $x \searrow 1/2$, where β is a system parameter and $C_\pm$ are positive constants. The structure of this map in the vicinity of $x=0$ is similar to that of the Pomeau-Manneville (PM) map near its indifferent fixed point. Actually, the left part ($x < 1/2$) of the map $\phi(x)$ is the same as that of the piecewise linear PM map analyzed by Tasaki and Gaspard [1]. On the other hand, the right part ($x > 1/2$) is different from that of the PM map in that the derivative of the map $\phi(x)$ is divergent as $x \searrow 1/2$. It is important that this singular structure makes the Lebesgue measure invariant under the transformation $\phi(x)$. It is also important that this system can be easily extended to 2-dimensional area-preserving map, whose dynamics of the expanding direction is given by the map $\phi(x)$. This relation between 1D map $\phi(x)$ and its area-preserving extension is the same as that between the Bernoulli and the baker transformations.

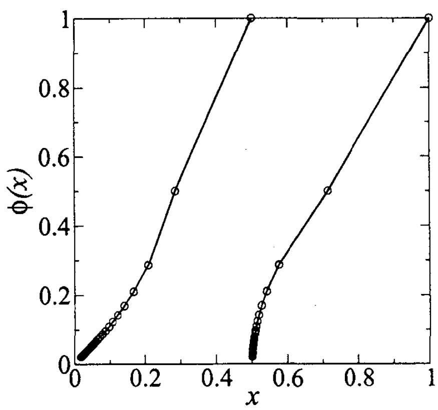

Figure 1: The piecewise linear map $\phi(x)$. The circles indicate endpoints of the linear segments.

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2 Numerical results

Fig. 2(a) shows power spectral densities $S(f)$ of time series $x(t)$ produced by the map $\phi(x)$ as $x(t+1) = \phi(x(t))$. They exhibit clear $1/f^{\gamma}$ scalings in a low frequency region; Fig. 2(b) displays the scaling exponent γ of the power spectrum $S(f) \sim 1/f^{\gamma}$ as a function of the exponent β. And I show the theoretical prediction by a dashed line in Fig. 2(b) which is obtained by calculating the eigenvalues of the Frobenius-Perron operator of the map $\phi(x)$ (The derivation will be reported elsewhere [2].). Obviously, the numerical results show a good agreement with the theoretical prediction.

It is worth noting that although these numerical calculations are for 1D map $\phi(x)$, this system can be extended to the 2D area-preserving map as discussed above; therefore this area-preserving extension of $\phi(x)$ has also long time correlations.

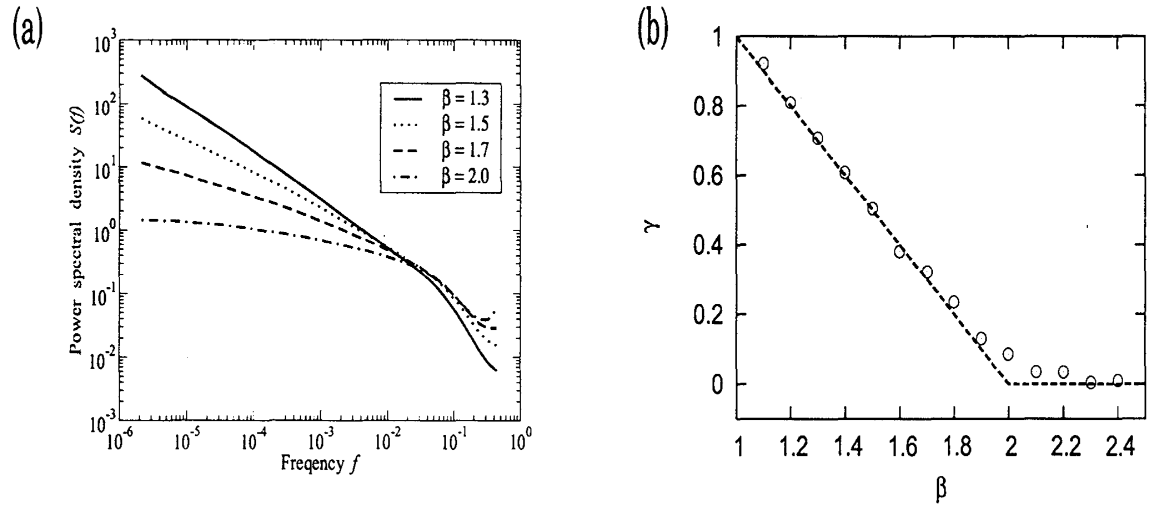

Figure 2: (a) The power spectral densities (PSDs) of $x(t)$ (log-log plot) for four different values of the exponent $\beta : \beta = 1.3,\ 1.5,\ 1.7,\ 2.0$. (b) The scaling exponent γ of the PSD $S(f) \sim 1/f^{\gamma}$ as a function of the exponent β. The circles are the numerical results obtained by least square fitting in a low frequency region (below $f = 10^{-4}$) of the PSDs $S(f)$; and the dashed line is the theoretical prediction.

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References

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[2] In preparation.